Math 121 2.1 Limits and Continuity

Objectives

1) Limits from graphs; left- and right-sided limits; limits are y-values.
2) Two reasons a limit does not exist
3) Limits from tables
4) Limits that can be found by direct evaluation
5) Limits that cannot be found by direct evaluation.
   - simplify by factoring and canceling a factor
6) Infinite limits (the answer is $+\infty$ or $-\infty$) and the weirdness that they don't exist.
7) Limits <u>at</u> infinity (the x-value goes to $+\infty$ or $-\infty$)
8) Finding a limit of one variable (even though there are two variables in the expression)
9) Continuous graphs, the definition of continuity
10) Graphs that are sometimes discontinuous; discontinuities
11) Relationship between limits and continuity.

Want more details?

Limits → Math 250 2.2 notes

Algebra for limits → Math 250 2.3 notes

Infinite Limits → Math 250 2.4 notes

Limits at Infinity → Math 250 2.5 notes

Continuity → Math 250 2.6 notes

on class website, go to Math 250 then Lecture Notes

A limit is the y-value the function approaches.
It may not be the actual value of the function.

$\lim_{x \to c} f(x) = L$ ← limit notation

A limit does not exist if

a) The y-value it approaches from the left is different from the y-value it approaches from the right

$\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x)$ ← one-sided limit notation

+ sign means from right of $x=c$

− sign means from left of $x=c$.

b) The y-value it approaches is ∞ or $-\infty$.

We can find limits

a) from tables
b) from graphs
c) from algebraic functions.

A graph is continuous if we can draw it without picking up the pencil. In math, this means

$\lim_{x \to c} f(x) = f(c)$ for all values of c.

This means

1) $\lim_{x \to c} f(x)$ exists

2) $f(c)$ is defined

3) $\lim_{x \to c} f(x) = f(c)$

① $f(x) = \dfrac{x^2+9x-36}{3x-9}$

a) Complete the table – round to 4 decimal places.

x	y
2.9	4.9667
2.99	4.9967
2.999	4.9997
3	not defined
3.001	5.0003
3.01	5.0033
3.1	5.0333

b) Find the limits from table and graph

$\lim_{x\to 3^+} f(x) = \boxed{5}$ The y-values approch 5 from the right.

$\lim_{x\to 3^-} f(x) = \boxed{5}$ The y-values approach 5 from the left.

$\lim_{x\to 3} f(x) = \boxed{5}$ The y-values approach 5 from both sides.

c) Find the limit by algebra:

$\lim_{x\to 3} \dfrac{x^2+9x-36}{3x-9}$ factor and cancel

$= \lim_{x\to 3} \dfrac{\cancel{(x-3)}(x+12)}{3\cancel{(x-3)}}$

$= \lim_{x\to 3} \dfrac{x+12}{3}$ Substitute $x=3$

$= \dfrac{3+12}{3} = \boxed{5}$

d) Is $f(x) = \frac{x^2+9x-36}{3x-9}$ continuous? $\boxed{\text{no.}}$ There's a hole at (3,5).

e) Write the intervals of continuity

$\boxed{(-\infty, 3) \cup (3, \infty)}$

it is continuous everywhere <u>except</u> $x=3$.

f) Where is f(x) discontinuous?

$\boxed{x=3}$

g) what is f(3)?

$\boxed{\text{undefined}}$

② The answers to a)-f) are exactly the same as for ①.

g) what is f(3)?

$\boxed{2}$ It has its own special piece in the piecewise function, just for $x=3$.

③ $f(x) = \frac{1}{3}x + 4$

a) $\lim\limits_{x \to 3^+} f(x) = \boxed{5}$

b) $\lim\limits_{x \to 3^-} f(x) = \boxed{5}$

c) $\lim\limits_{x \to 3} f(x) = \boxed{5}$

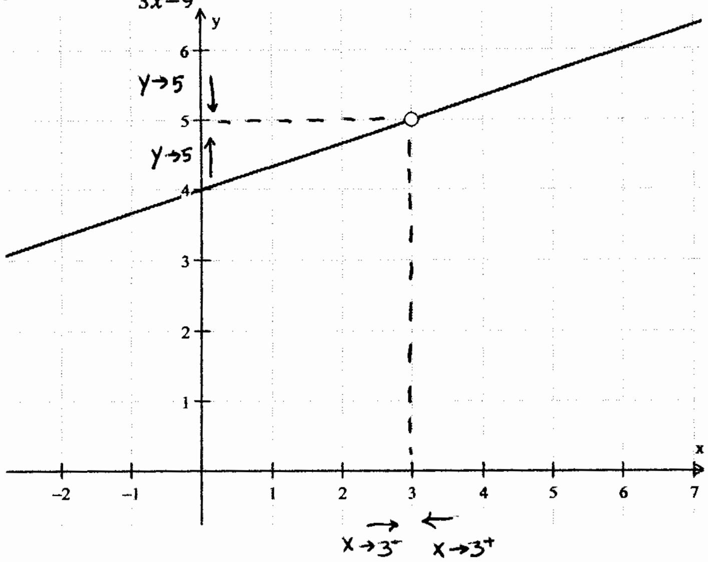
d) continuous? $\boxed{\text{yes}}$

e) intervals? $\boxed{(-\infty, \infty)}$

f) discontinuities $\boxed{\text{none}}$

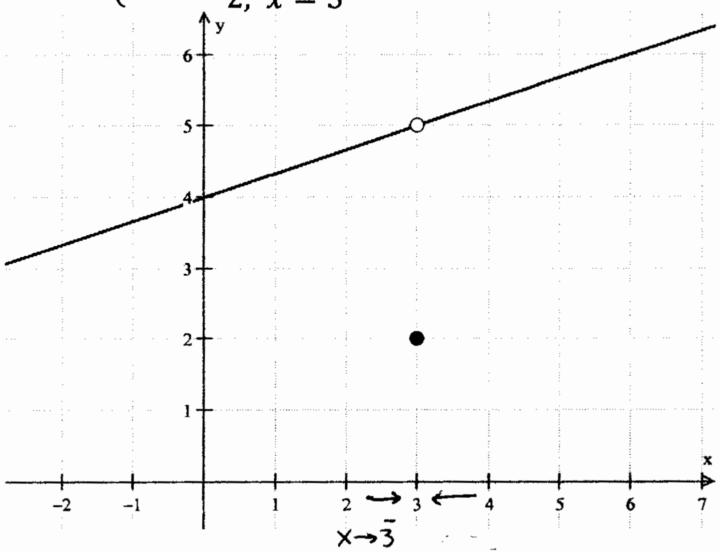
g) $f(3) = \frac{1}{3}(3) + 4 = \boxed{5}$.

1) $f(x) = \frac{x^2+9x-36}{3x-9}$



g) f(3) is undefined.

2) $f(x) = \begin{cases} \frac{x^2+9x-36}{3x-9}, & x \neq 3 \\ 2, & x = 3 \end{cases}$

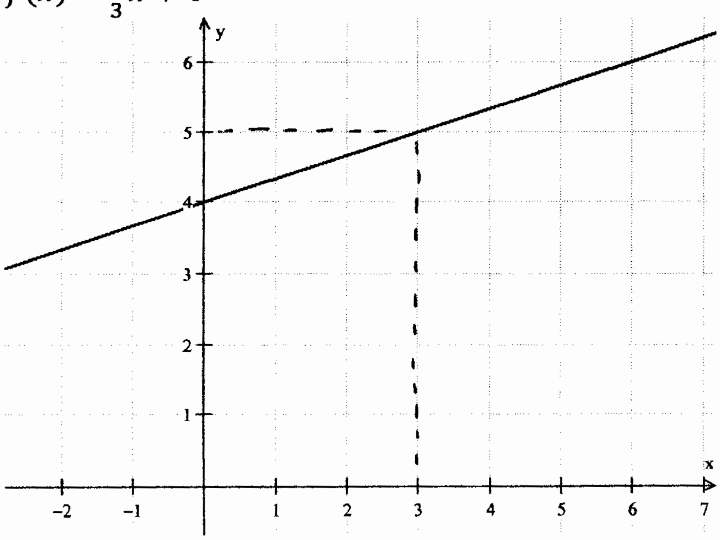


None of the work changes for 2.
Just as in 1

- Table is the same
- Limits are the same
- Continuity & discontinuity are the same.

What's different? only
g) f(3) = 2 in 2

3) $f(x) = \frac{1}{3}x + 4$



This function is continuous.

$\lim_{x \to 3} f(x) = f(3) = 5.$

4) $f(x) = \begin{cases} \frac{1}{3}x+4 & x \leq 3 \\ \frac{1}{3}x+2 & x < 3 \end{cases}$

a) Complete the table

x	y	
2.9	4.9667	use $\frac{1}{3}x+4$
2.99	4.9967	
2.999	4.9997	
3	5	
3.001	3.0003	use $\frac{1}{3}x+2$
3.01	3.0033	
3.1	3.0333	

b) Find the limits.

$\lim_{x \to 3^+} f(x) = \boxed{3}$

$\lim_{x \to 3^-} f(x) = \boxed{5}$

$\lim_{x \to 3} f(x) = \boxed{\text{does not exist}}$

c) Find the limit by algebra

$f(3) = \frac{1}{3}(3)+4 = 5$ So $\lim_{x \to 3^-} f(x) = \boxed{5}$

other piece $\frac{1}{3}(3)+2 = 3$ so $\lim_{x \to 3^+} f(x) = \boxed{3}$

d) Continuous? $\boxed{\text{no}}$

e) Intervals of continuity? $(-\infty, 3) \cup (3, \infty)$

f) Discontinuities? $\boxed{x=3}$

g) $f(3) = \boxed{5}$

(5) $f(x) = \begin{cases} \frac{1}{3}x+4 & x<3 \\ 4 & x=3 \\ \frac{1}{3}x+2 & x>3 \end{cases}$

a) Table $\Rightarrow$ same as (4)

b) limits? $\Rightarrow$ same as (4)

c) algebra? $\Rightarrow$ same as (4)

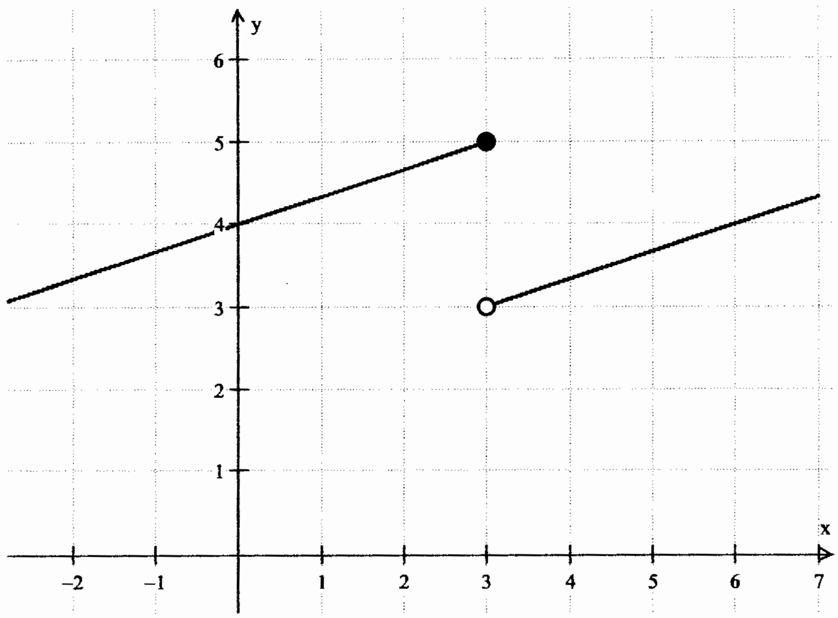
d) continuous? $\Rightarrow$ same as (4)

e) intervals? $\Rightarrow$ same as (4)

f) discontinuity? $\Rightarrow$ same as (4)

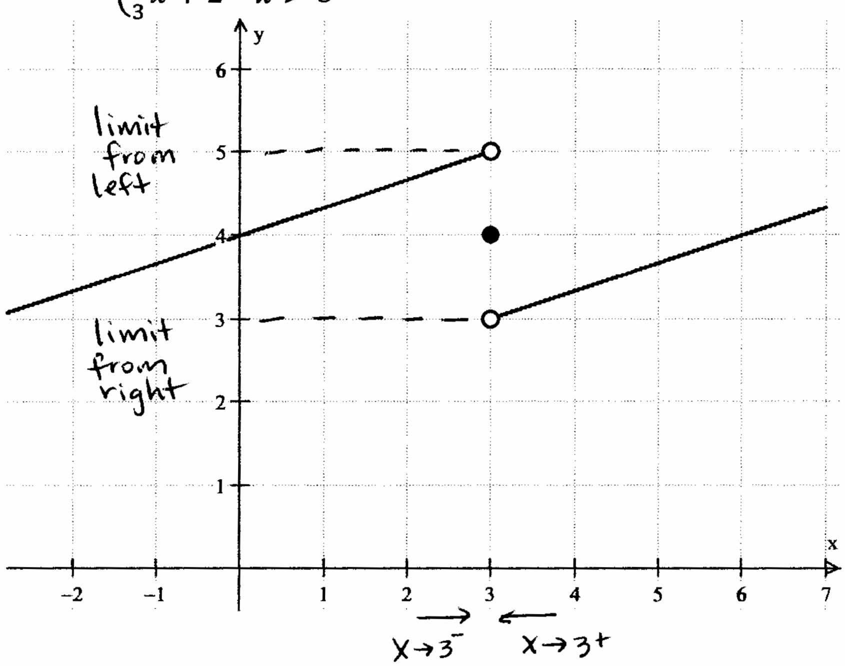
g) $f(3) = \boxed{4}$.

4) $f(x) = \begin{cases} \frac{1}{3}x + 4, & x \le 3 \\ \frac{1}{3}x + 2, & x > 3 \end{cases}$



$f(3) = 5$ (colored dot)

5) $f(x) = \begin{cases} \frac{1}{3}x + 4 & x < 3 \\ 4 & x = 3 \\ \frac{1}{3}x + 2 & x > 3 \end{cases}$



$\lim_{x \to 3^+} f(x) = \boxed{3}$

$f(3) = 4$ (colored dot)

6 $f(x) = \frac{1}{(x-3)^2}$

a) $\lim_{x \to 3^+} f(x) = \boxed{+\infty, \text{ does not exist}}$

b) $\lim_{x \to 3^-} f(x) = \boxed{+\infty, \text{ does not exist}}$

c) $\lim_{x \to 3} f(x) = \boxed{+\infty, \text{ does not exist}}$

vertical asymptote at $x=3$

d) continuous? $\boxed{\text{no}}$

e) intervals? $\boxed{(-\infty, 3) \cup (3, \infty)}$

f) discontinuity? $\boxed{x=3}$

g) $f(3) = \boxed{\text{not defined}}$

h) $\lim_{x \to \infty} f(x) = \boxed{0}$ off the right edge of graph

i) $\lim_{x \to -\infty} f(x) = \boxed{0}$ off the left edge of graph

j) describe asymptotes

horizontal asymptote $y=0$

⑦ $f(x) = \frac{2x-4}{x-3}$

a) $\lim_{x\to 3^+} f(x) = \boxed{+\infty\text{, does not exist}}$

b) $\lim_{x\to 3^-} f(x) = \boxed{-\infty\text{, does not exist}}$

c) $\lim_{x\to 3} f(x) = \boxed{\text{does not exist}}$

d) continuous? $\boxed{\text{no}}$

e) intervals? $\boxed{(-\infty, 3), (3, \infty)}$

f) discontinuity? $\boxed{x=3}$

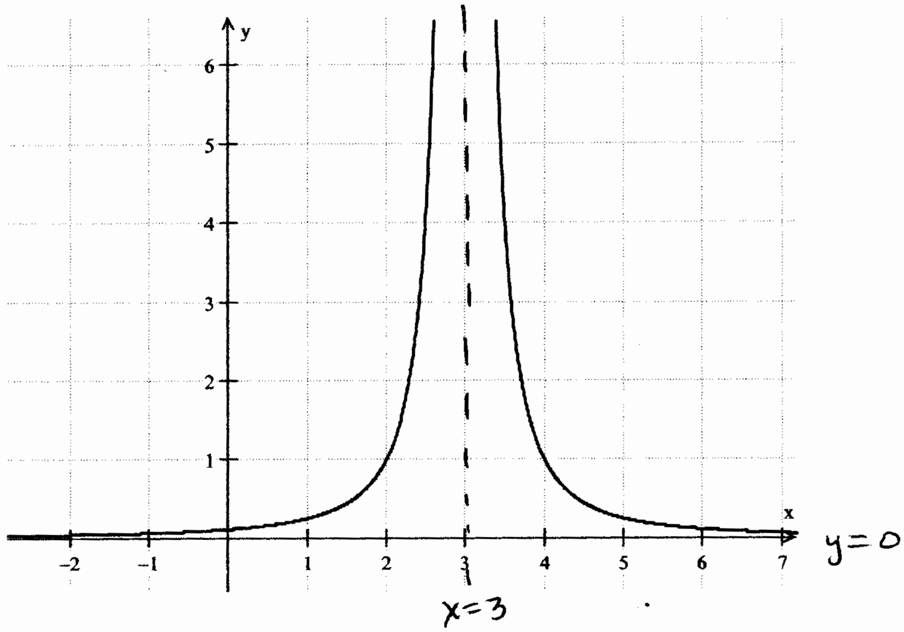
g) $f(3) = \boxed{\text{not defined}}$

h) $\lim_{x\to\infty} f(x) = \boxed{2}$

i) $\lim_{x\to -\infty} f(x) = \boxed{2}$

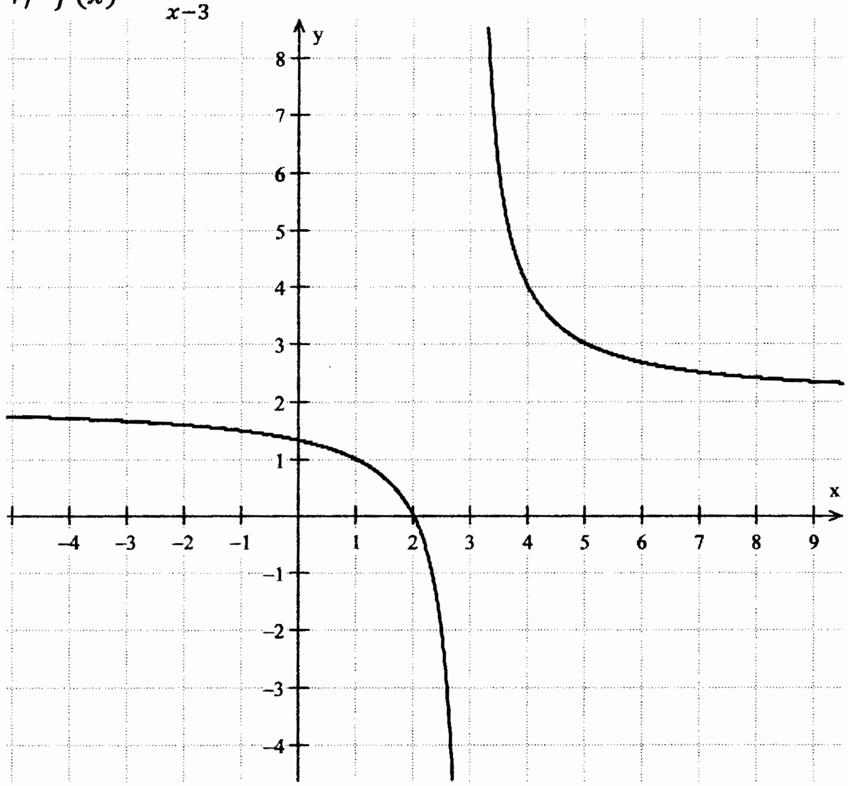
j) describe asymptotes: $\boxed{\begin{array}{l}\text{vertical } x=3 \\ \text{horizontal } y=2\end{array}}$

6) $f(x) = \frac{1}{(x-3)^2}$



f(3) undefined

7) $f(x) = \frac{2x-4}{x-3}$



Find the limits.

(8) $\lim_{x\to 3} \frac{x^2-3x}{x^2+4x+4}$

$= \lim_{x\to 3} \frac{x(x-3)}{(x+2)(x+2)}$

$= \frac{3(3-3)}{(3+2)(3+2)}$

$= \frac{3\cdot 0}{5\cdot 5}$

$= \boxed{0}$

(9) $\lim_{x\to 3} \frac{x^2-3x}{x^2-}$

$= \lim_{x\to 3} \frac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

$= \lim_{x\to 3} \frac{x}{x+3}$

$= \frac{3}{3+3}$

$= \frac{3}{6}$

$= \boxed{\frac{1}{2}}$

(10) $\lim_{h\to 0} \frac{x^2h-xh^2+h^3}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(x^2-xh+h^2)}{\cancel{h}}$

$= \lim_{h\to 0} (x^2-xh+h^2)$

$= x^2 - x(0) + 0^2$

$= \boxed{x^2}$

(11) $\lim_{h\to 0} \frac{5x^4h-9xh^2}{h}$

$= \lim_{h\to 0} \frac{x\cancel{h}(5x^3-9h)}{\cancel{h}}$

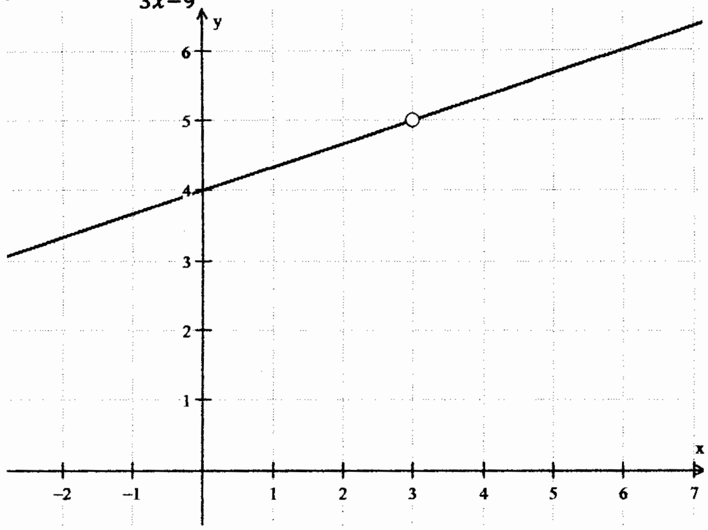
$= \lim_{h\to 0} x(5x^3-9h)$

$= x(5x^3-9\cdot 0)$

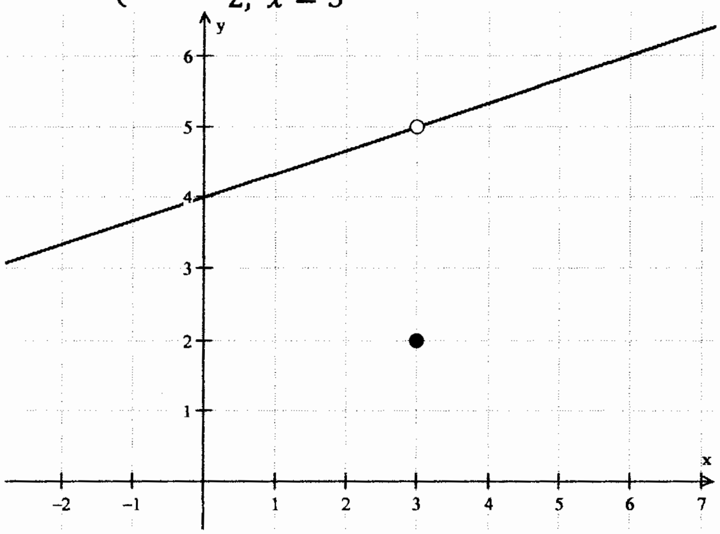
$= x(5x^3)$

$= \boxed{5x^4}$

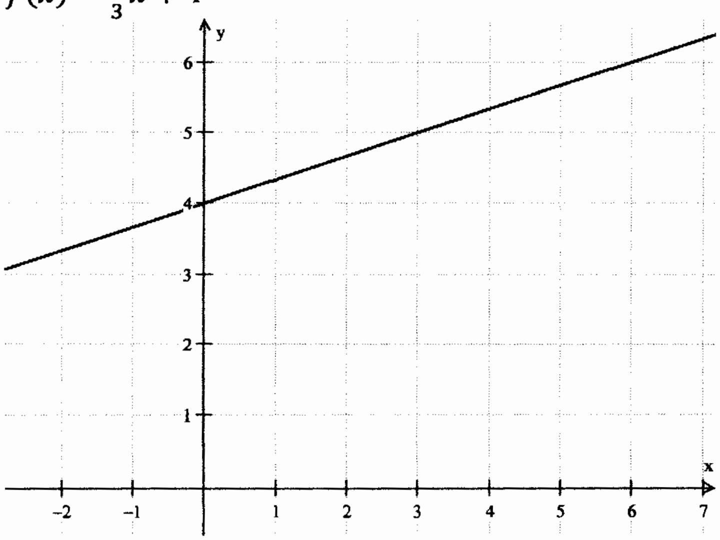
1) $f(x) = \frac{x^2+9x-36}{3x-9}$



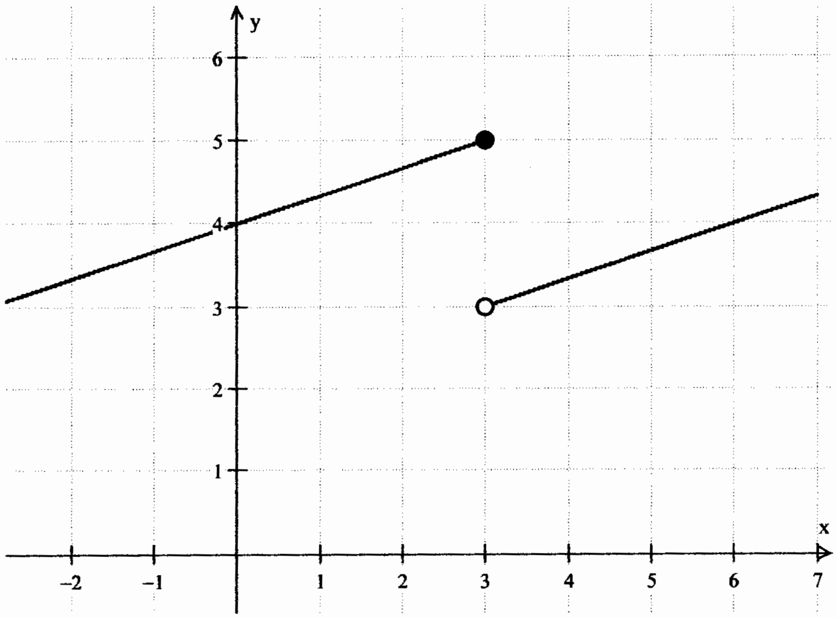
2) $f(x) = \begin{cases} \frac{x^2+9x-36}{3x-9}, & x \neq 3 \\ 2, & x = 3 \end{cases}$



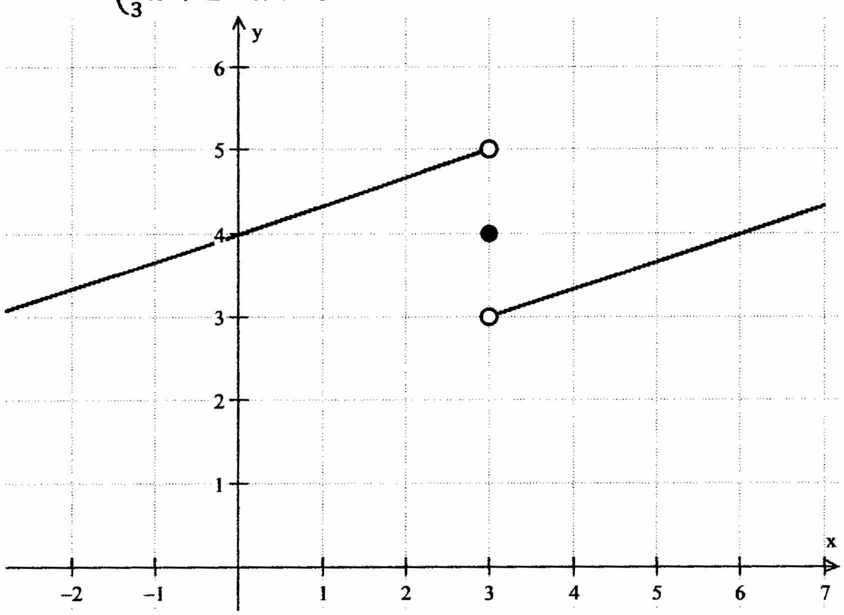
3) $f(x) = \frac{1}{3}x + 4$



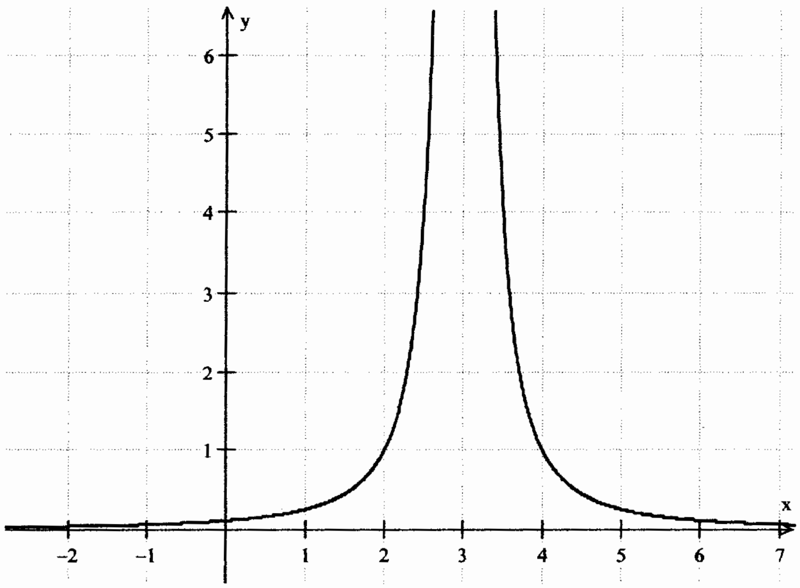
4) $f(x) = \begin{cases} \frac{1}{3}x + 4, & x \leq 3 \\ \frac{1}{3}x + 2, & x > 3 \end{cases}$



5) $f(x) = \begin{cases} \frac{1}{3}x + 4 & x < 3 \\ 4 & x = 3 \\ \frac{1}{3}x + 2 & x > 3 \end{cases}$



6) $f(x) = \frac{1}{(x-3)^2}$



7) $f(x) = \frac{2x-4}{x-3}$